Working Model Technical Note: Modeling Friction in Pin Joints

Keith Reckdahl
Knowledge Revolution
66 Bovet Road, Suite 200
San Mateo, CA 94402

1-800-766-6615

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Summary

The friction in pin joints is often small enough such that neglecting it does not adversely affect the accuracy of a dynamic simulation. However, in some cases, friction in pin joints has a substantial effect on the motion, and it must be included in static and dynamic analysis.

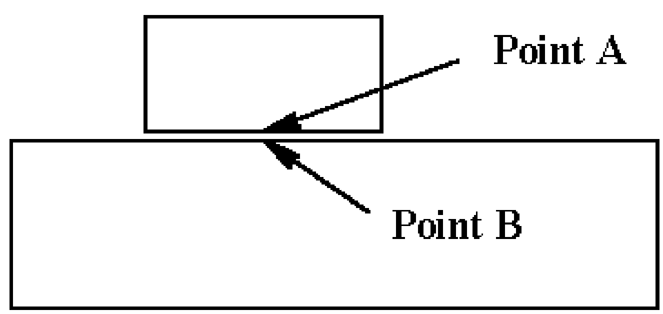
This document describes how the use of Working Model's formula language allows a user to accurately model friction pin joints and thus increase the reliability of the simulation.

Classical Friction Model

When two points are in contact, the classical friction model breaks the friction force analysis into two cases:

1) When the points are moving relative to each other, the surfaces are said to be slipping and the friction is called *kinetic friction*.
2) When the surfaces have no relative motion, the surfaces are said to be sticking and the friction is called *static friction*.

Figure 1
Two points in contact



Let A and B be two points which are in contact (see Figure 1). If A and B are sliding, the classical friction model approximates the friction force on point A from point B as:

$$\vec{F}_f^{A/B} = -\mu_k N \frac{\vec{V}^{A/B}}{\left\| \vec{V}^{A/B} \right\|} \qquad \text{(Equation 1)}$$

where:

μ_s is the *kinetic coefficient of friction*

N is the magnitude of the normal force between A and B

$\vec{V}^{A/B} = \vec{V}^{A} - \vec{V}^{B}$ is the velocity of A relative to B

and $\vec{V}^{A}$ and $\vec{V}^{B}$ are the velocities of points A and B in the global reference frame. (Actually, any reference frame may be used, provided that the same reference frame is used for both point A and point B.)

If points A and B are sticking, the classical friction model approximates the friction force as the force necessary to keep A and B in contact provided that:

$$\left\| \vec{F}_f^{A/B} \right\| \leq \mu_s N \qquad \text{(Equation 2)}$$

If the required force is greater than μ_s N, then slipping begins with

$$\vec{F}_f^{A/B} = -\mu_s N \vec{v} \qquad \text{(Equation 3)}$$

where **v** is the unit vector in the direction of impending motion of A with respect to B

An Alternate Friction Model

Disadvantages of the Classic Friction Model

It is important to note that the classical friction model described by equations 1-3 is only an approximation of the actual friction phenomenon. The classical friction model is hard to implement for the following reasons:

1. There are two equations for friction: one when the surfaces are sticking and one when the surfaces are slipping. The static-to-slipping and slipping-to-static transitions can significantly decrease the speed of numerical integration.
2. When the surfaces are sticking, the constraint force keeping the surfaces from slipping must be calculated.
3. During transition from sticking to slipping, the direction of impending motion must be calculated.

An Alternate Friction Model

To avoid the above disadvantages, Arun Banerjee of Lockheed Palo Alto Research Center along with Paul Mitiguy and Keith Reckdahl developed an alternate friction model that produces accurate results while allowing faster numerical integration.

When the coefficient of static friction μ_s is close to the coefficient of kinetic friction μ_k, the friction force can be approximated well by a single equation:

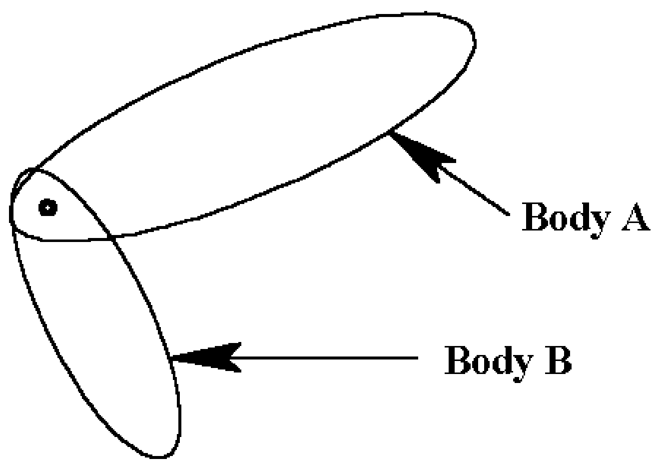
$$\vec{F}_f^{A/B} = -m\,N \frac{\vec{V}^{A/B}}{\left\|\vec{V}^{A/B}\right\| + e} \qquad \text{(Equation 4)}$$

where ε is a positive number which is small relative to the typical values for $\vec{V}^{A/B}$. (See the "Choosing a Value for ε" section below)

The use of a single coefficient does not introduce much error for two reasons:

1. The values for μ_k and μ_s are usually fairly close to each other
2. Although the motion of a dynamic system may be sensitive to changes in μ_k, the motion of a dynamic system is generally insensitive to changes in μ_s. Thus using equation 4 with $\mu=\mu_k$ generally provides good results.

Figure 2
Two Bodies Connected by a Rotational Joint



Rotational Friction

When A and B are two bodies connected by a rotational joint (see Figure 2) a rotational equivalent to equation 4 can be constructed:

$$\vec{T}_f^{A/B} = -Rm\,N \frac{\vec{w}^{A/B}}{\left\|\vec{w}^{A/B}\right\| + e} \qquad \text{(Equation 5)}$$

where:

- μ = coefficient of friction
- N = magnitude of constraint force produced by joint
- R = joint radius (radius at which sliding is occurring)
- $\vec{w}^{A/B}$ = is the angular velocity of A in B
- ε = a positive number which is small relative to the typical values for $\vec{w}^{A/B}$. See the "Choosing a Value for ε" section below.

Often equation 5 is rewritten as:

$$\vec{T}_f^{A/B} = -m_{eff}\ N \frac{\vec{w}^{\,A/B}}{\left\| \vec{w}^{\,A/B} \right\| + e} \qquad \text{(Equation 6)}$$

where $\mu_{eff} = R\mu$ is the *effective coefficient of friction* for the pin joint.

Other Types of Joints

This rotational friction model assumes a sliding contact within the joint. Other joints (such as ball bearings) can also be modeled by obtaining a value for μ_{eff} through testing. Ideally, such testing also distinguishes the joint's viscous damping from its sliding friction.

This model also makes it easy to model of friction in joints which involve other forces. For example, a joint with a rotational spring, rotational damping, and friction would be modeled as:

$$\vec{T}_f^{A/B} = -kq^{A/B}\vec{v} - c\vec{w}^{\,A/B} - m_{eff}\ N \frac{\vec{w}^{\,A/B}}{\left\| \vec{w}^{\,A/B} \right\| + e} \qquad \text{(Equation 7)}$$

where
c = rotational damping coefficient
k = rotational spring constant
$\vec{v}$ is a unit vector parallel to the joint axis
$q^{A/B}$ is the rotation angle of A with respect to B

Examining equation 7, note that the first term is the torque produced by the rotational spring, the second term is the torque produced by the rotational damper, and the third term is the torque produced by the joint friction.

Choosing a Value for e

In the previous sections, ε was described as: "a positive number which is small relative to the typical values of the velocity." A convenient method for determining ε is:

1. **Let $V_{typical}$ be a "typical value" for the velocity. The smaller this velocity is, the more accurate the friction model will be but the longer the numerical integration will take.**
   To determine a "typical value," you must have some knowledge of the physical system. If you do not know how the system will move, choose an arbitrary "typical value" and run the simulation. Use the results of the simulation to choose a more-accurate "typical value" and re-run the simulation.

2. **Calculate the value for ε by**
   $$e = V_{typical} * 1.0e - 6$$
   This should produce accurate results without excessive integration time. Make the 1.0e-6 smaller for more accurate friction modeling, or make it larger for easier integration.

Determining the Coefficient of Friction

The coefficient of friction is usually determined by one of the following methods:

1. Testing
2. Friction Tables in a reference book (such as the CRC handbook). Since friction is very dependent on surface finish, the numbers from friction tables are only approximate unless they specify surface finish data.

In general, testing produces a more accurate estimate of the coefficient of friction than do friction tables. Since changes in temperature and humidity can greatly change the coefficient of friction, even testing cannot accurately determine the actual coefficient of friction.

Instead of trying to determine a single value for the coefficient of friction, it is better to determine lower and upper bounds for the friction. Multiple simulations must then be run using different values of friction.

Procedure for Modeling Friction

The following steps outline the procedure for modeling friction in a pin joint. These steps must be performed for each joint which includes friction.

Note: The term "select" (as in "select the meter") means to click on the arrow tool and then click on the object. When an object is selected, its corners become highlighted. If you click on the object and its corners do not become highlighted, it is not selected and you should try clicking on another part of the object or by enclosing it with a Selection Rectangle. For example, some controls (inputs) can only be selected by clicking on the control's name, not the control itself.

The phrase "deselecting all objects" means to click on the arrow too and then click on the background.

For information on selecting objects, see pages 202-203 of the Working Model User's Manual.

General Procedure for Modeling Friction in Pin Joints

1. **Determine the effective coefficient of friction for the joint.**
   a. **Determine the coefficient of friction μ. If you have values for both μ_k and μ_s, use $\mu = \mu_k$.**
   b. **Determine the radius of the joint**
   c. **Calculate** $\mu_{eff} = R * \mu$

2. **Choose a value for Choose a "typical speed" for ε as described in the "Choosing a Value for ε" section.**

3. **Draw bodies in Working Model. Use the Motor tool to connect any bodies joined with a frictional joint. (Do not use the pin joint tool to connect the bodies.)**

4. **Each joint connects two mass objects: the “top object” and the “base object.” At the joint, the top object overlaps the base object. Determine the object number (such as mass[2] or constraint[4]) for:**
   a. **the top object**
   b. **the base object**
   c. **the motor which connects the objects**

   Note that placing the cursor over the object causes the object number to be displayed on the help ribbon at the top of the screen. (If the help ribbon is not displayed, choose “Workspace” and “Help Ribbon” from the “View” menu.)

5. **Perform the following steps to create an input box for the effective coefficient of friction:**
   a. **Deselect all objects and create a Generic Control (Input) by choosing “New Control” from “Define” menu.**
   b. **Select the control icon and choose “Properties...” from the “Window” menu**
   c. **In the “Properties” window, click on “Text Box” and close “Properties” window.**
   d. **Select the control icon and choose “Appearance” from the “Window” menu**
   e. **In the “Window” menu, change “Generic Control” to “Coef of Friction” and close “Appearance” window.**
   f. **Click on the text portion of the “Coef of Friction” text box and enter effective coefficient of friction calculated in step #1.**

6. **Perform the following steps to create a Meter to calculate friction torque:**
   a. **Create a meter by selecting the top object and then choosing “Velocity” and “Rotation graph” from the “Measure” menu.**
   b. **Click twice on the arrow in the upper left corner of the meter to convert it to text output.**
   c. **Select the meter and choose “Properties...” from “Window” menu**
   d. **Edit Properties Window to calculate equation #6.**

      For example, the Properties Window in Figure 3 calculates equation #6 when:

      Mass[1] is the Base Object
      Mass[2] is the Top Object
      Constraint[8] is Joint Motor
      Input[11] is “Coef of Friction” text box
      Output[12] is Meter

      Notes:

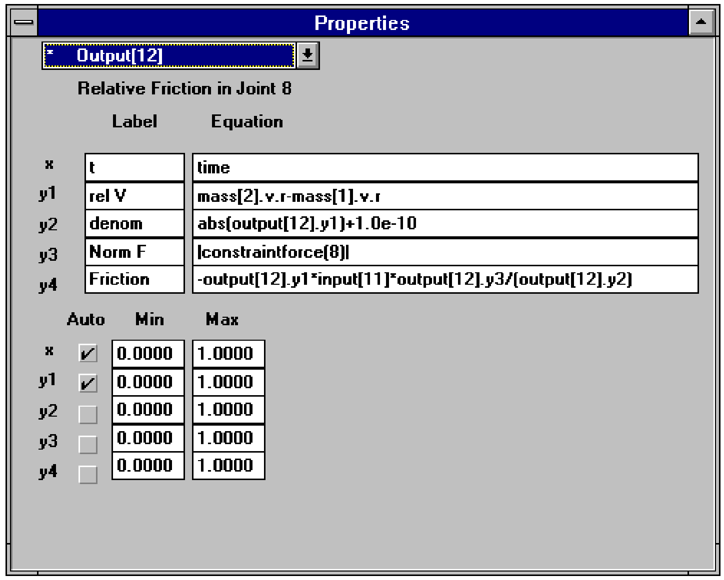
      output[12].y1: Calculates the angular velocity of mass[2] with respect to mass[1]

      output[12].y2: Calculates the denominator in equation 6 when ε is chosen to be 1.0e-10.

      output[12].y3: Calculates the magnitude of the constraint force produced by the joint (constraint[8])

      output[12].y4: Using lines y1-y3 of the meter and the coefficient of friction text box (input[11]), this line calculates the friction torque specified by equation 6.

Figure 3
Properties Window for Meter



7. **Perform the following steps to make the torque motor produce the torque calculated in step #5**
   a. **Select the torque motor and choose "Properties..." from the "Window" menu**
   b. **In the Properties Window, choose "Torque" from the "Type" menu**
   c. **In the Properties Window's "Value" box, specify which meter cell calculated the frictional torque.**

      For the example in Figure 3,
      Output[12].y4
      would be entered in the "Value" box.
   d. **Close Properties Window**

8. **Repeat steps 3-6 for each joint with friction**

9. **Run the Working Model simulation by clicking on the "Run" button.**